# ZONAL WIND ERRORS IN THE BAROTROPIC MODEL

C. L. BRISTOR

Joint Numerical Weather Prediction Unit, U.S. Weather Bureau, Suitland, Md.*

[Manuscript received September 11, 1958; revised January 20, 1959]

ABSTRACT

Zonal wind errors in a series of numerical barotropic forecasts are discussed. A characteristic error is the growth of west winds in middle latitudes and their decay to the south. Momentum transport in the absence of momentum sources and sinks in the barotropic model is suggested as the principal contributor to this type of error. Evidence of nonlinear growth of the zonal wind error suggests that use of past errors as a simple linear correction of forecast zonal profiles is not a promising solution of the problem particularly when no provision is made for stabilization of the longest waves. In an attempt to throw some light on the remaining question of what can be done to improve the barotropic model with respect to zonal wind errors, a series of experimental forecasts is made using a form of the barotropic vorticity equation with a persistent frictional term that provides crudely for momentum sources and sinks. Although the results are, to a considerable degree, inconclusive, they suggest that future efforts should be directed toward the inclusion of friction in a more realistic atmospheric model.

## 1. INTRODUCTION

Since October 1957, barotropic forecasts have been produced by the Joint Numerical Weather Prediction (JNWP) Unit on an octagonal grid covering most of the Northern Hemisphere and extending south of the 500-mb. zone of maximum westerlies in all regions. Prior to this time forecasts were produced on a grid having an extreme boundary problem which tended to compromise the analysis of other types of systematic errors [1]. Since boundary problems have been appreciably reduced with the octagonal grid, the resolution of the remaining types of systematic errors has become a more rewarding task. A persistent retrogression of the longest wave components, particularly in the subtropics, was brought into sharper focus as the largest systematic error. An empirical remedy for this type of error was discussed by Wolff [2] and a less restrictive remedy by Cressman [3].

Another systematic error is immediately apparent when forecasts are verified from the standpoint of the zonal wind. One observes a decrease of west winds to the south in the subtropics and an increase in temperate latitudes.

The purpose of this paper is to present a detailed discussion of the zonal wind errors of a series of 24-, 48-, and 72-hour barotropic forecasts made on a daily basis from October 4, 1957 to May 31, 1958. Also, some interpretation of the errors is attempted and some experiments aimed at providing the best empirical correction are discussed.

## 2. ZONAL ERRORS

Several diagnostic programs available in the JNWP Unit have made possible the accumulation of an appreciable amount of data describing the behavior of routinely produced barotropic forecasts. One such program produces mean monthly algebraic height error information over the entire grid from 24-, 48-, and 72-hr. forecasts. Further processing yields the resulting zonal wind error patterns. Figure 1 presents this information for the individual months. Here one sees the marked tendency for the forecasts to shift the maximum westerlies to the north. The same general type of error is repeated month after month suggesting a highly systematic error-producing mechanism. Figure 2 presents a day-to-day latitudinal distribution of 48-hr. forecast errors. On occasion the error pattern is interrupted but by and large the same characteristic error pattern appears day after day.

Phillips [1] has suggested momentum transport as the principal contributor to this type of error. In the isobaric system of coordinates (c.f. [4]), combination of the equations of continuity and $x$-component of motion gives

$$\frac{\partial u}{\partial t}+\frac{\partial (uu)}{\partial x}+\frac{\partial (uv)}{\partial y}+\frac{\partial (u\omega)}{\partial p}-fv+g\frac{\partial z}{\partial x}=0 \tag{1}$$

where the symbols have their conventional meanings. If equation (1) is integrated around a latitude circle and throughout the depth of the atmosphere the second and sixth terms drop out. The resulting equation in integral form can be regarded as the equation for zonal momentum per unit mass:

$$\frac{\partial}{\partial t}\int_o^{2\pi}\int_o^{p_o} ud\lambda dp+\frac{\partial}{\partial y}\int_o^{2\pi}\int_o^{p_o} uvd\lambda dp+\int_o^{2\pi}(u\omega)_{p_o}d\lambda=0 \tag{2}$$

where $\lambda$ is longitude. The last term enters only at the

*Present affiliation: Meteorological Satellite Section, U.S. Weather Bureau.

[1] Personal communication by Prof. N. A. Phillips during his visit as consultant to JNWP Unit, November 1957. Some aspects of the forecast investigations stemmed from discussions with him during this visit.

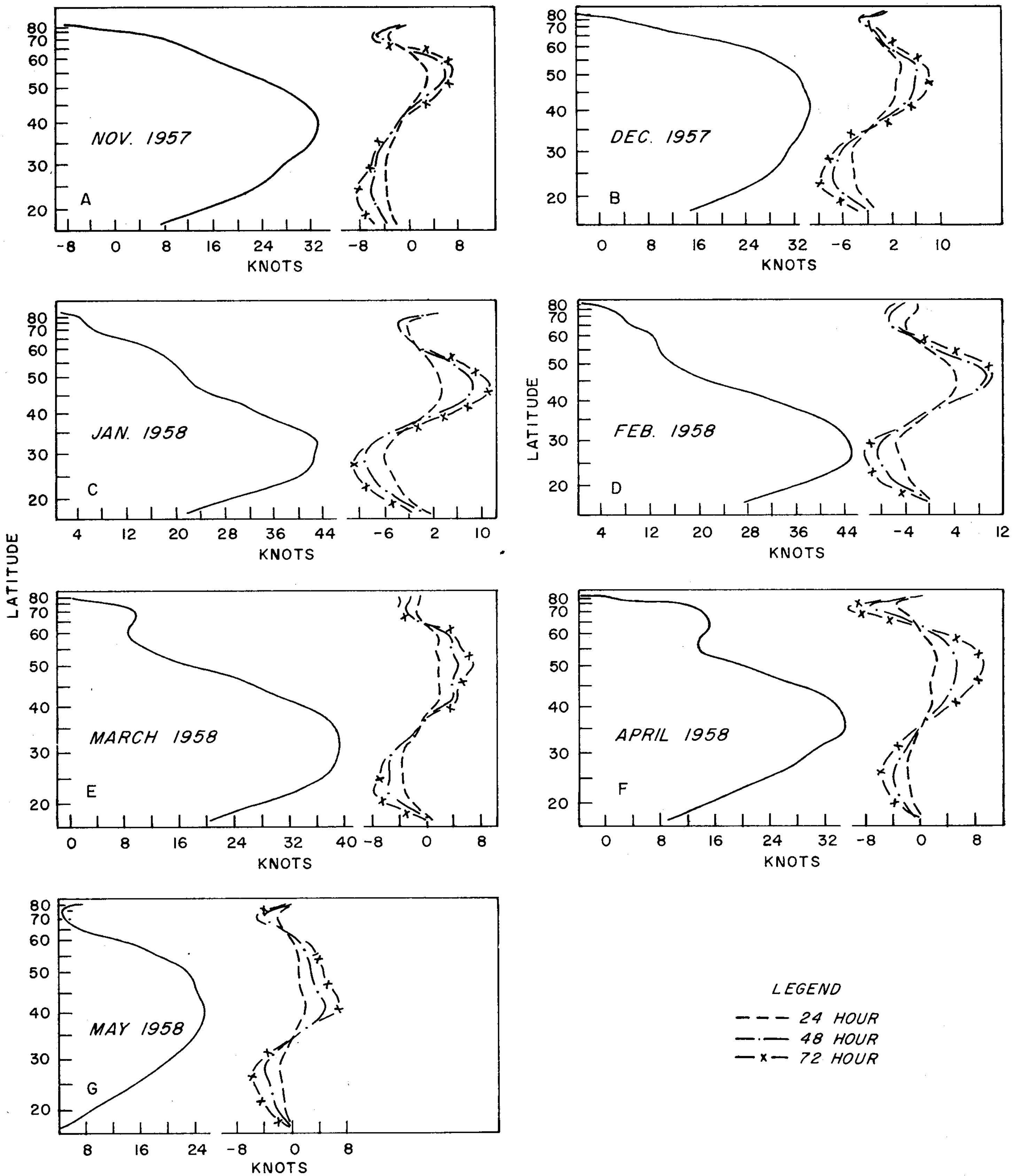


Figure 1.—Monthly average 500-mb. zonal wind profiles and average errors of barotropic forecasts. Beginning on April 10 all forecasts were altered by stabilizing the long waves.

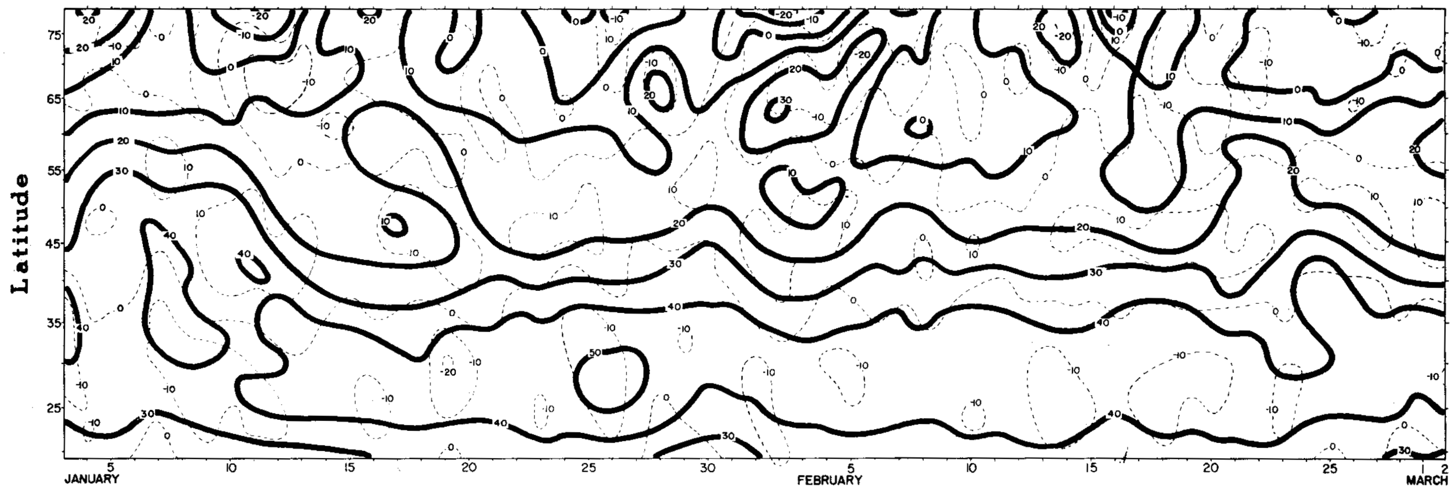


Figure 2.—Daily 500-mb. zonal wind profiles (kt.) for January and February 1958 (heavy lines) and corresponding errors of 48-hour barotropic forecasts (dashed lines).

pressure level near the surface (1000 mb.) since $\omega$ is assumed to vanish at the top of the atmosphere. There is thus an implied balance between the horizontal transport and vertical transport (source and sink) terms. From actual winds Buch [5] showed that the maximum northward transport of momentum at 500 mb. occurs near the latitude midway between the low level easterlies (momentum source) in the subtropics and the temperate latitude westerlies (momentum sink). It seems likely that the behavior of the barotropic model with regard to zonal wind errors reflects this same process, the momentum transport being a function of the orientation (tilt) of troughs and ridges, as noted earlier by Starr [6], and the observed errors thereby arising from the lack of momentum sources and sinks in the model.

It is interesting to examine some of the day-to-day fluctuations from this standpoint. A search through the 500-mb. maps for October 1957 indicated that the orientation of troughs south of 35° N. on October 22 was least favorable for the northward transport of momentum, but on the 26th was particularly favorable. These maps are presented in figure 3. The zonal wind errors (in knots) in the 48-hr. forecasts from these days are inserted in figure 3 over Mexico and the western States. It is noteworthy that the zonal wind errors south of 35° N. are strikingly different in the two cases, the depletion of west winds

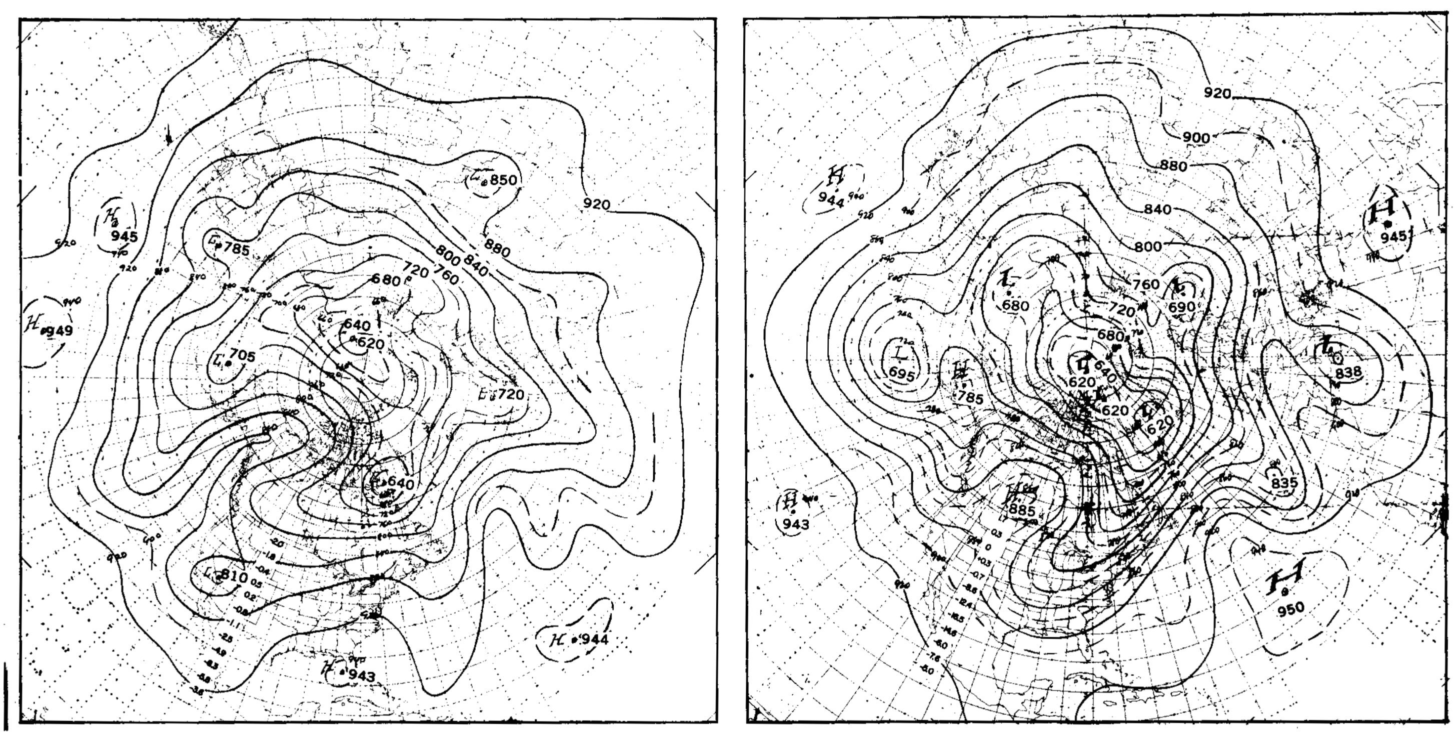

Figure 3.—Initial 500-mb. patterns and zonal wind error (listed near 105° W.) of the resulting 48-hour barotropic forecasts. (A) 0000 GMT, October 22, 1957, an example of a major northwest-southeast trough; (B) 0000 GMT, October 26, 1957, an example of a major northeast-southwest trough.

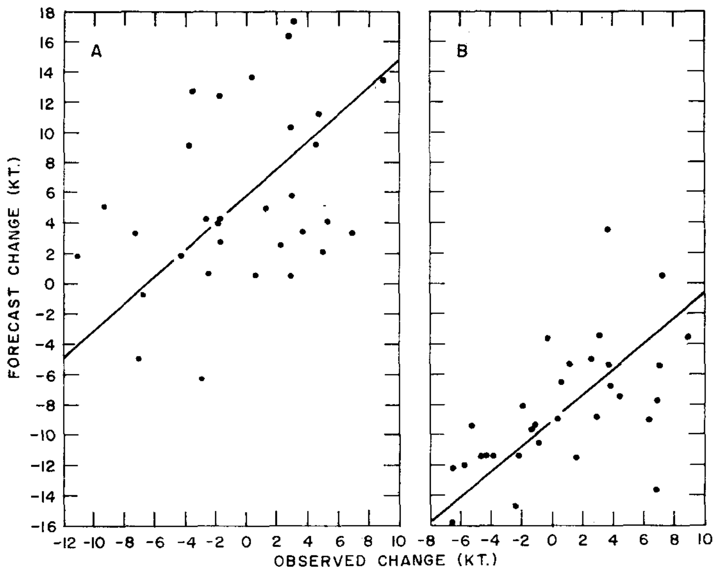


Figure 4.—48-hour forecast zonal wind changes plotted against observed zonal wind changes, January 1958. (A) 42° N., (B) 30° N.

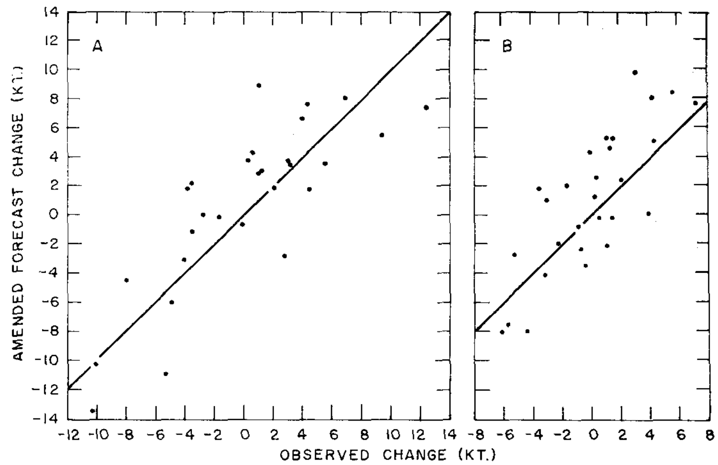


Figure 5.—48-hour forecast zonal wind changes (amended using fig. 4) plotted against the observed change, February 1958. (A) 42° N., (B) 30° N.

in the subtropics being almost negligible in the October 22 case and more than twice the monthly average in the October 26 case.

From the above discussion it is obvious that, even in the short-range forecast problem, fundamental questions of the general circulation are important. Clearly, any highly realistic atmospheric model must contain a momentum budget mechanism even for forecasts of a day or two. Short of models with such a physical mechanism there is a question of what might be done semi-empirically to improve less elegant models such as the currently operational barotropic model.

Before we make empirical corrections to the zonal profile, it is prudent to ask how much skill is contained in the forecasts. Figure 4 shows 48-hr forecast zonal wind changes versus corresponding observed zonal wind changes at two arbitrary latitude points along the profile. Figure 4A is for latitude 42° N. where the forecast west wind was in excess of the observed and figure 4B is for 30° N. where the reverse relation existed. Although the scatter of points is appreciable, definite skill beyond that of a persistence forecast is evident. Figure 5 indicates appreciable success of the operationally produced forecasts at the same latitudes for the following month (February) when they are modified by the curves of figure 4. The February forecasts were modified by entering them on the appropriate ordinate of figure 4 and reading the modified forecasts from the abscissa of the January curve. Here improvement over persistence is obvious.

In many respects January and February circulations were much alike as indicated in the similarity of zonal profiles and errors. One might therefore question the utility of such relationships for other months. However, the zonal error patterns do not show abrupt changes from month to month as do the zonal profiles themselves. For example, the crest of maximum west wind in the zonal profile changed abruptly from a weaker than normal and north of normal pattern in December to a stronger than normal and south of normal picture in January. During this same period the mean 24-hr. zonal wind error profile did not undergo any shift in phase and changed only slightly in amplitude and general shape.

Evidence of nonlinear growth of zonal wind error is also apparent, particularly in figure 1A through E. Such a situation would tend to hamper any attempt to use evidence of past errors as a simple linear persistence type correction. The retrograde long waves, an improper feature of the prediction model [2], may interact to enhance such nonlinearity. This possibility is suggested by the more nearly linear behavior of the errors in figure 1F where two-thirds of the forecasts have been altered by stabilizing the long waves and in figure 1G where all forecasts have been thus altered. Without stabilization of the long waves, forecast troughs were characteristically skewed by moving too slowly at low latitudes and too fast at high latitudes. Such skewing suggests possible augmentation of the northward transport of momentum in the forecasts. The stabilization technique (to be discussed in section 3) effectively forecasts persistence of that portion of the motion contained in the longest waves. Unfortunately no extended record of stabilized and unstabilized forecasts was available for comparison in this study.

## 3. EXPERIMENTAL FORECASTS WITH FRICTION

After the above diagnosis of zonal wind errors there remains the question of what can be done to improve the barotropic model in this respect. N. A. Phillips[2] has suggested the addition of momentum sources and sinks by means of a friction term involving the zonal wind in the layer of turbulent mixing. In practice, if such a model

[2] See footnote 1 on p. 57.

is to remain a single parameter model, the wind in the lower layer must be derived from 500-mb. information. Preliminary calculations by Phillips revealed such a derived wind to be unsatisfactory. However, there remains the possibility that the zonal wind profile within the turbulent mixing layer changes rather slowly so that it may provide beneficial, persistent, momentum source and sink regions throughout the forecast computation. The assumption that momentum sources and sinks can be provided in the equations of motion by a term proportional to the negative of the wind velocity near the surface in the layer of turbulent exchange yields the vorticity equation in the form

$$\frac{\partial \zeta}{\partial t} = -\mathbf{V} \cdot \nabla \zeta - \beta v - K\zeta_0 \tag{3}$$

where $\zeta$ is relative vorticity, $\mathbf{V}$ is the wind vector, $\beta$ is the Rossby parameter, $K$ is a coefficient involving the turbulent exchange of momentum between the earth's surface and the free air through convective eddies, and all quantities refer to the level of nondivergence (assumed to be 500 mb.) except where subscript $o$ designates a surface value.

A barotropic forecast code was modified to include a full field of the third term on the right of equation (3). This field suitably scaled, was created using the relation

$$\zeta_0 = -\frac{\partial \overline{U}_0}{\partial y}, \tag{4}$$

where $\overline{U}_0$ is the zonal profile of the initial wind near the surface. This term was added as a persistent correction at each hourly time step. Such a zonally symmetric correction must obviously be harmful over some areas on occasion even though the over-all effect is beneficial because, in general, the zonal wind profile does not change a like amount in all longitudinal sectors.

Six comparative forecasts have been made with this procedure in order to judge its effect in several different synoptic situations. Other comparisons were also made by testing a corrective profile based on a time average during the forecast rather than on persistence and using 850-mb. rather than lower-level winds. As a whole these comparisons were inconclusive. Generally the modified forecasts improved the zonal profile in the subtropics but caused further deterioration about as often as improvement at middle and higher latitudes. The assumption of persistence and the choice of wind level in the corrective term seemed to be of only secondary importance. At this stage there seemed to be little point in more tests along this line. The implied balance between momentum source and sink regions and horizontal transport from the standpoint of a simple continuity argument does not appear to be a particularly useful concept when applied on a daily basis. In other words, the compensation for momentum transport at 500 mb. does not seem to be particularly direct.

Table 1.—*Root mean square height errors in feet for four forecast techniques. See text for explanation of models*

| Initial Date | Forecast Model | | | |
|---|---|---|---|---|
| | a | b | c | d |
| Jan. 12, 1958 | 212 | ------- | 229 | 411 |
| Jan. 14, 1958 | 256 | 313 | 252 | 414 |
| Apr. 14, 1958 | 246 | 245 | 269 | (*) |
| Apr. 16, 1958 | 186 | 178 | 176 | |
| May 18, 1958 | 165 | 160 | 170 | |

* Not available after April 10, 1958.

The concept of a simple correction thus seems to be reduced to the idea of a more or less stereotyped correction which would remove the systematic error from a large group of forecasts. One way to accomplish this would be merely to impose the initial zonal wind profile as a persistent component in the forecast. Such a technique would preclude realization of inherent skill in predicting the zonal profile as suggested in the scatter diagrams of figures 4 and 5 and as suggested by the slowly changing monthly error profiles. The alternate method would involve the use of equation (3) but with the last term simulated by a mean corrective profile (ideally a concurrent one) as discussed above. The difficulty, of course, still involves the forecast of the mean error profile. Between the drawback of the persistence zonal forecast and the drawback of having to project the mean profile error ahead for several weeks, no choice can be made short of an extended series of tests. Five additional test cases of this type were carried out. Results of two of the five cases are displayed in figures 6 and 7 and are discussed later from the standpoint of zonal errors. Although such a sample is very limited, it is interesting to compare forecasts having zonal corrections with forecasts from the simple non-divergent barotropic model. Apart from zonal wind verification there remains a question of repercussions in other types of verification statistics. Table 1 indicates the root mean square height errors for several competing forecasts in each case.

Column (a) gives the scores of a model somewhat similar to that described in connection with equation (3) except that the long wave components have been stabilized by the inclusion of a fourth Jacobian term. This technique, devised by L. P. Carstensen,[3] uses an additional constant Jacobian field at each time step to stabilize the longest wave components. This Jacobian field, applied in the reversed sense, is merely the advection of relative vorticity of a space-smoothed initial field. The zonal profile term is added as described above except that it is now produced in each case from the vorticity of the 500-mb. mean zonal wind error profile from the preceding month. Column (b) presents results from a modified barotropic code, devised by Wolff [2], which imposes a persistence forecast of the zonal profile as well as waves one, two, and three. The technique which produced the results in column (c) is similar to that of (b) in that long waves

[3] Unpublished.

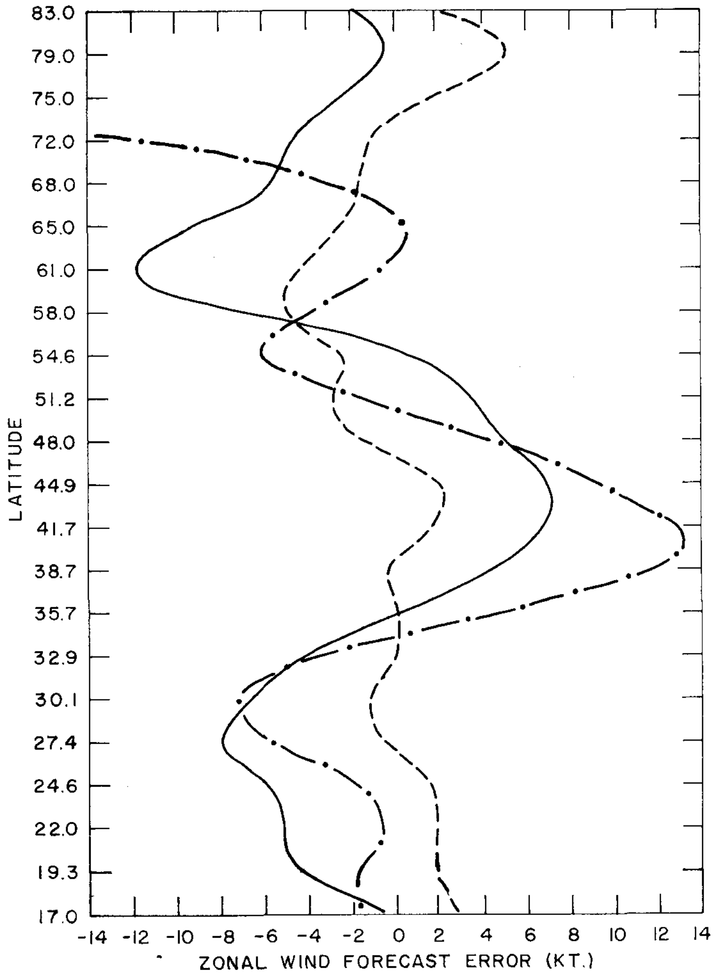


Figure 6.—Comparison of zonal wind errors for 48-hour forecasts from 0000 GMT, January 12, 1958 and produced by the non-divergent operational barotropic model (solid line), a modified model with long waves stabilized and a zonally symmetric correction (dashed line), and by persistence (dash-dot line).

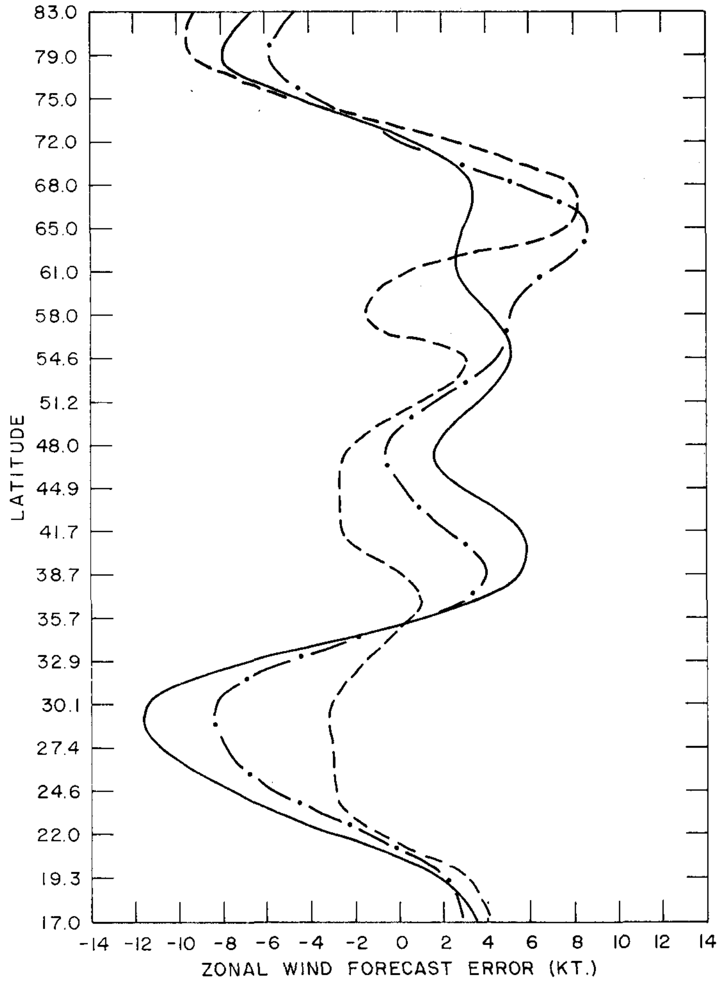


Figure 7.—Comparison of zonal wind errors for 48-hour forecasts produced by the same methods as in figure 6 from 0000 GMT, May 18, 1958.

are stabilized, but the zonal profile is not altered. Column (d) presents simple non-divergent barotropic forecast scores.

In view of the small sample size and the difference in long wave treatment there appears to be little difference between the root mean square height errors of the first three methods when considering 48-hr. forecasts. There is perhaps a slight indication that both methods for correcting the zonal errors would provide an average improvement in root mean square height error. (Two cases extended to 72 hours by P. M. Wolff indicate substantial improvement of method (b) over method (c).) From the standpoint of verification of zonal values, method (b) would obviously be less successful for cases with rapidly changing profiles. As shown in figure 2, forecasts from 0000 GMT January 12, 1958 involve this problem. The zonal errors of methods (a), (b), and (c) for this case are presented in figure 6. Here a persistence forecast has a damaging effect whereas method (a) seems to behave satisfactorily. Figure 7 presents the case from 0000 GMT May 18, 1958 which displayed a rather cut-up pattern of zonal wind changes (the negative of the persistence curve). Here persistence proved to be the better forecast although method (a) was, in general, an improvement over method (c).

## 4. CONCLUSIONS

Barotropic forecasts on a hemispheric scale contain an error in the zonal wind which is highly systematic in the sense that the belt of maximum westerlies is moved too far to the north. The local 48-hr. zonal wind error on either side of the maximum westerlies amounts to about 20 percent of the maximum zonal wind. Inspection of individual cases where troughs have extreme tilt lends considerable support to the suggestion that these errors are related to momentum budget deficiencies of the model. Experiments with modified barotropic forecast codes indicate the possibility of improving the forecasts from the standpoint of zonal winds through the application of a zonally symmetric correction based upon an average of past errors.

Less improvement is obtained in similar experiments which utilize a zonal friction term obtained from the initial surface map.

In either case any improvement from the zonal standpoint is usually obtained at the expense of harmful results in some local areas—the advantage thereby being a question of usage of the prognostic chart. In view of this impasse it would seem that future efforts should be directed toward the inclusion of friction in a more realistic model.

## REFERENCES


1. G. P. Cressman and W. E. Hubert, "A Study of Numerical Forecasting Errors," *Monthly Weather Review*, vol. 85, No. 7, July 1957, pp. 235–242.
2. P. M. Wolff, "The Error in Numerical Forecasts Due to Retrogression of Ultra-Long Waves," *Monthly Weather Review*, vol. 86, No. 7, July 1958, pp. 245–250.
3. G. P. Cressman, "Barotropic Divergence and Very Long Atmospheric Waves," *Monthly Weather Review*, vol. 86, No. 8, Aug. 1958, pp. 293–297.
4. R. C. Sutcliffe, "A Contribution to the Problem of Development," *Quarterly Journal of the Royal Meteorological Society*, vol. LXXIII, No. 317–318, July/Oct. 1947, pp. 370–383.
5. H. S. Buch, "Hemispheric Wind Conditions During the Year 1950," Massachusetts Institute of Technology, General Circulation Project, Final Report Part 2, contract AF 19(122)–153, May 31, 1954, 126 pp.
6. V. P. Starr, "An Essay on the General Circulation of the Earth's Atmosphere," *Journal of Meteorology*, vol. 5, No. 2, 1948, pp. 39–43.